Inventory Management Systems with Hazardous Items of Two-Parameter Exponential Distribution

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Abstract. We study the inventory system with two-parameter exponential distributed hazardous items in which production and demand rate are constant. The mathematical model is developed with an exponential distribution hazardous item to obtain the total cost per unit time of an inventory system. It is illustrated with the help of numerical example. The inventory controlled systems in terms of first order differential equations are solved numerically. The effect of changes in the model parameters on decision variables and average total cost of an inventory system are studied through numerical example.

Key words: Inventory-Production Model, Storage, Exponential distributed hazardous Item, Operation Research and Management Science, Performance measure.

AMS Subject Classification. 90B30, 90B05, 49J15, 58E25.

1. The Introduction

Inventory may be considered as an accumulation of a product that would be used to satisfy future demands for that product. It requires a policy to control that inventory. Since the pioneering work of Harris (1913) inventory models are being treated by mathematical techniques. Until the nineteen fifties only inventory models with deterministic demands were considered. After the nineteen fifties, due to developments within the theory of stochastic processes, the research on inventory models with stochastic demands initiated. In all inventory models a general assumption is that products generated have indefinitely long lives. In general, almost all items deteriorate over time. There are many other products in the real world that are subject to a significant rate of deterioration. The impact of product deterioration should not be neglected in the decision process of production lot size. Deterioration can be classified as age-dependent on-going deterioration and age-independent on-going deterioration. Deteriorating inventory models have been widely studied in recent years. Ghare and Schrader (1963) were two of the earliest researchers to consider continuously decaying inventory for a constant demand. Shah and Jaiswal (1977) presented an order-level inventory model for deteriorating items with a constant rate of deterioration. Aggarwal (1978) developed an order-level inventory model by correcting and modifying the error in Shah and Jaiswal's analysis (Shah and Jaiswal, 1977) in calculating the average inventory holding cost. Covert and Philip (1973) used a variable deterioration rate of two parameter Weibull distribution to formulate a model with the assumptions of a constant demand rate and no shortages. Choi and Hwang (1986) developed a model determining the production rate for deteriorating items to minimize the total cost function over a finite planning period. Raafat (1985) extended the model, given in Park (1983) to deal with a case in which the finished product is also subject to a constant rate of decay. Yang and Wee (2003) considered a multi-lot-size production-inventory system for deteriorating items with constant production and demand rates. This type of inventory control problems have been studied also by Sugapriya and Jeyaraman (2008a, 2008b); Shah and Acharya (2008); Mishra and Shah (2008); and Sabahno (2008).

In this paper we study the inventory system with two-parameter exponential distributed hazardous item in which production and demand rate are constant. An exponentially decaying inventory was first developed by Ghare and Schrader (1963). The certain commodities were observed to shrink with time by a proportion which can be approximated by a negative exponential function of time. Here, the novelty we take into consideration is that a constant hazardous rate followed by the two-parameter exponential distribution. The probability density function of a two-parameter exponential distribution is given by

$$f(t;\mu,\theta)=\frac{1}{\theta}\exp\left\{-\left(t-\mu\right)/\theta\right\}, \qquad t\geq\mu \quad \text{and} \quad t\geq 0,$$

where μ is the location parameter, $\theta>0$ is the scale parameter. The probability distribution function is

$$F(t;\mu,\theta)=1-\exp\left\{-\left(t-\mu\right)/\theta\right\}.$$

The hazardous rate of the on-hand inventory is given by

$$H\left(t;\mu,\theta\right)=\frac{f(t;\mu,\theta)}{1-F(t;\mu,\theta)}=\frac{1}{\theta}, \quad t\geq 0, \quad \theta>0.$$

We are concerned with an inventory system for a single product subject to two-parameter exponential hazardous item to determine a production cycle time in this study. The objective of this paper is to develop a mathematical model for obtaining an optimal purchase quantity for constant hazardous item associated with exponential distribution during the cycle time. To minimize the total cost per unit time of an inventory-production system is also our interest. The effect of changes in the model parameters on decision variables and total cost of an inventory-production system is studied through numerical example.

This paper is organized as follows. In section-2 basic assumptions and notations are employed for the development of the inventory model. In section 3 we develop a mathematical model for optimal purchase quantity in the inventory-production system. An illustrative numerical example and final conclusions of the results are presented in the subsequent sections.

2. Basic Model Assumptions and Notations

The following are the assumptions applied in the proposed model:

(i) A finite planning horizon is assumed.
(ii) The production unit of the product is available and it can meet the demand.
(iii) The inventory system deals with single item.
(iv) The demand rate for the product is known and constant.
(v) Shortage is not allowed.
(vi) The hazardous of units in inventory follows an exponential distribution

The notations that are employed in this study:

u : units of the production rate per unit time.
y : actual demand of the product per unit time.
$H(t;\mu,\theta)=\frac{1}{\theta}$: a constant hazardous rate per unit time.
A : set up cost per order.
$h>0$: inventory holding cost coefficient per unit time.
$x(t)$: inventory level of the product.
k : production cost per unit time.
$x(0)$: initial inventory.

3. Development of the Model and Solutions

The objective of the inventory problem here is to determine the optimal order quantity so as to keep the total relevant cost as low as possible. Let $x(t)$ be the on-hand inventory at any instant of time $t\ (0\le t\le T)$. The state level of inventory amplify due to production rate and depletes due to demand rate and hazardous rate $\frac{1}{\theta}$ of exponential distribution. The dynamics of the state equation of inventory level $x(t)$ of the product at time t over period $[0,T]$ is governed by the differential equation

$$dx(t)=\left[u-y-\frac{1}{\theta}x(t)\right]dt,\quad 0\le t\le T \qquad (3.1)$$

with boundary conditions $x(0)=0$ and $x(T)=0$.
The solution of differential equation (3.1) using boundary equation $x(T)=0$ is

$$x(t)=x(0)\exp\left(-\frac{1}{\theta}t\right)+\int_0^t[u-y]\exp\left(-\frac{1}{\theta}(\tau-t)\right)d\tau,\quad \text{for all } t\in[o,T],$$

from which we have

$$x(t)=[u-y]\theta\left[1-\exp\left(\frac{1}{\theta}(T-t)\right)\right],$$

and assuming $\theta<1$, an approximate value is obtained by neglecting those terms of degree greater than or equal to 2 in θ using Tailors series expansion of the exponential functions,

$$x(t)=[u-y]\left[(T-t)-\frac{(T-t)^2}{2\theta}\right]. \qquad (3.2)$$

The inventory holding cost in the system during production cost per unit time is

$$IHC = \frac{1}{T}\int_0^T x(t)\,dt$$

$$= \frac{1}{T}h\left[u-y\right]\int_0^T\left[\left(T-t\right)-\frac{\left(T-t\right)^2}{2\theta}\right]dt$$

$$= \frac{1}{6\theta}hT\left[u-y\right]\left[3\theta-T\right] \qquad (3.3)$$

Using $x(0)=\tilde{x}$ and by (3.2) we obtain

$$\tilde{x}(t)=\left[u-y\right]\left[T-\frac{T^2}{2\theta}\right].$$

The number of units that hazarded; $H(T)$ during one cycle time is given by

$$H(T)=\tilde{x}(t)-\left[u-y\right]T=\left[u-y\right]\left[-\frac{T^2}{2\theta}\right].$$

So, cost of hazardous rate per unit time is

$$CH=\frac{k}{T}\left[u-y\right]\left[-\frac{T^2}{2\theta}\right]. \qquad (3.4)$$

The production cost per unit time is given by

$$PC=\frac{uk}{T}. \qquad (3.5)$$

The set up cost per unit time is given by

$$SC=\frac{A}{T}. \qquad (3.6)$$

Therefore by (3.3), (3.4), (3.5) and (3.6) the average total cost per unit time is given by

$$K(T)=\frac{1}{T}\left[IHC+CH+PC+SC\right]$$

$$=\frac{u-y}{2\theta}\left[h\left(1-T/3\theta\right)-k\right]+\frac{uk}{T^2}+\frac{A}{T^2}. \qquad (3.7)$$

Our objective is to minimize the total cost per unit time $K(T)$. The necessary condition for total cost $K(T)$ to be minimum is $\frac{\partial K}{\partial T}=0$ and $\frac{\partial^2 K}{\partial T^2}>0$ for all $T>0$.

Therefore we obtain

$$\frac{\partial K}{\partial T}=-\frac{\left(u-y\right)h}{6\theta^2}-\frac{2uk}{T^3}-\frac{2A}{T^3}\le 0,$$

and

$\frac{\partial^2 K}{\partial T^2}=\frac{6}{T^4}\left[uk+A\right]>0$ for all $T>0$ i.e., the second derivative is found to be positive.

4. Numerical Illustrations

Consider an inventory system with the following parametric values in the proper units: A=$50/set up, u=70 units/unit time, y=30 units/unit time, k=$15/unit time, h=3 unit/unit time, $H(t;\mu,\theta)=0.08$, $\theta=\dfrac{1}{H(t;\mu,\theta)}=12.5$, T=4.7081 unit time. Then from (3.3), (3.4), (3.5), (3.6) and (3.7) we obtain IHC=$247.02, CH=-$112.99, PC=$223.01, SC=$10.62 and K(T)=$78.09 respectively.

The inventory level $x(t)$ in terms of the first-order differential equation (3.1) is solved numerically using the mathematical package MATLAB version 6.5. For this, figure-1 shows the convergence of the optimal inventory with unit time towards inventory goal level $\hat{x}(t)$ (say) about 3.25.

Now we are going to display the Inventory holding cost, cost of hazardous rate and average total cost per unit time which are computed for the set of hazardous rates. Figures from 2 to 5 show the inventory holding cost, cost of hazardous rate and total cost with the reference of hazardous rates given below:

Hazardous rate					
$H_1(t;\mu,\theta)$	$H_2(t;\mu,\theta)$	$H_3(t;\mu,\theta)$	$H_4(t;\mu,\theta)$	$H_5(t;\mu,\theta)$	$H_6(t;\mu,\theta)$
0.06	0.08	0.1	0.125	0.15	0.175

Figure-2 shows that when hazardous rate increases, inventory holding cost per unit time increases at a certain level (say $247.02 per unit time) and then it gradually decreases. Figure-3 reveals that when hazardous rate increases, cost of hazardous rate per unit time decreases. Figure-4 shows that when hazardous rate increases, average total cost per unit time increases at a certain level (say $78.13 per unit time) then it starts declining. Figure-5 reveals that when hazardous rate increases and inventory holding cost increases, average total cost per unit time increases at a certain level (say $78.13 per unit time) then it abruptly decreases.

Figure-1

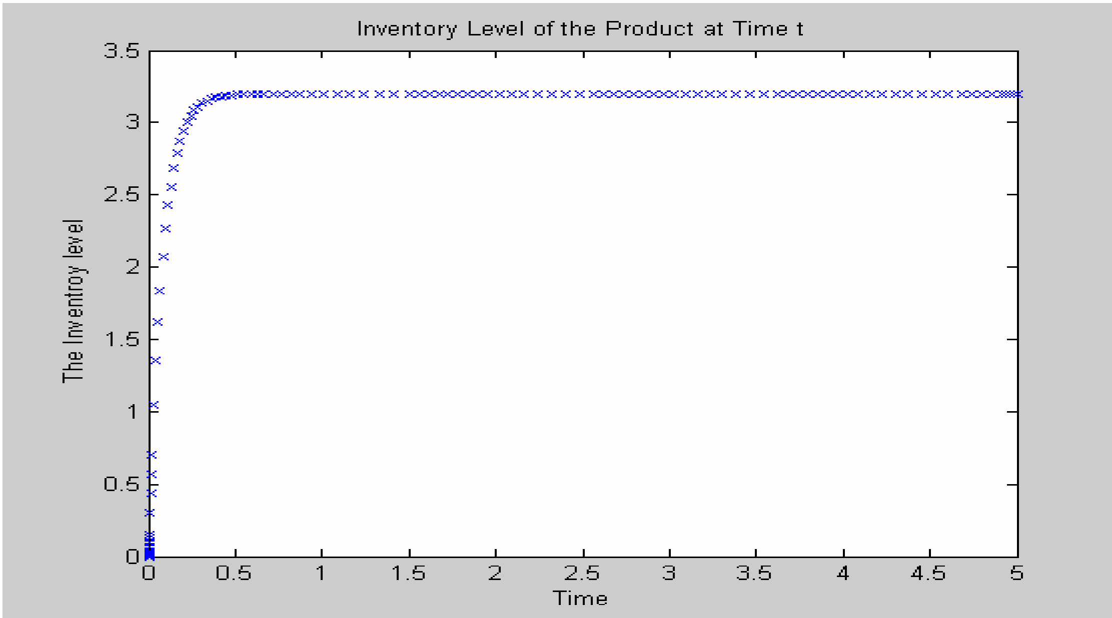


Figure-2

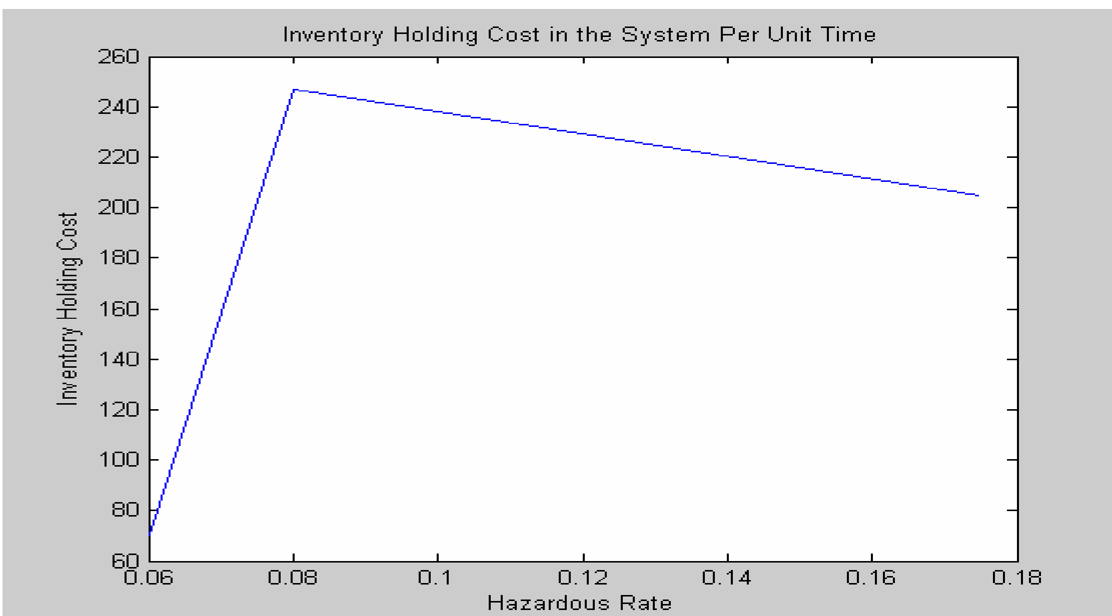


Figure-3

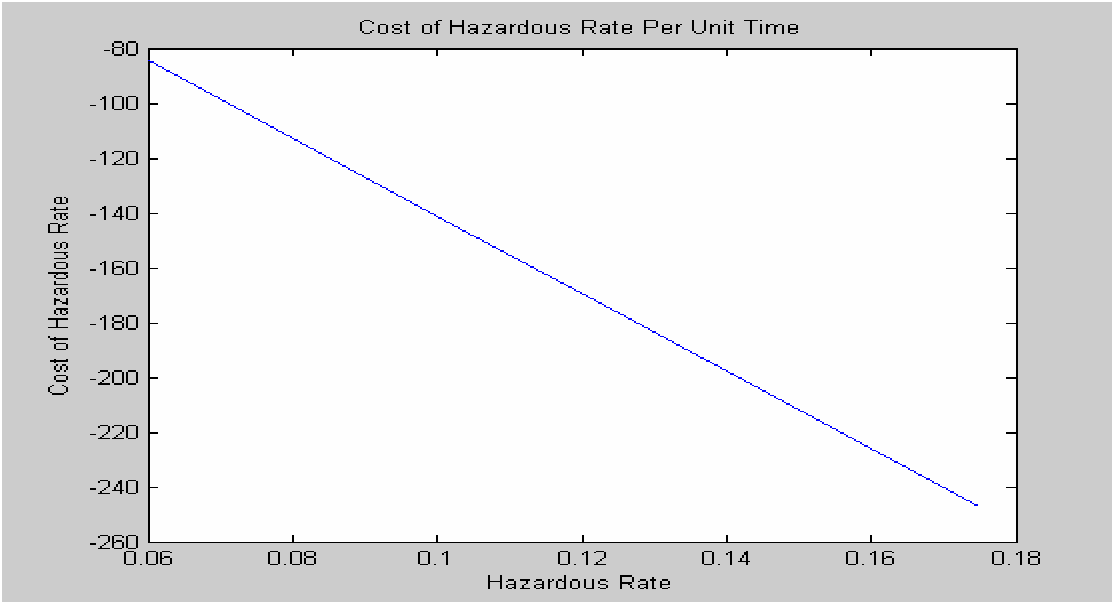


Figure-4

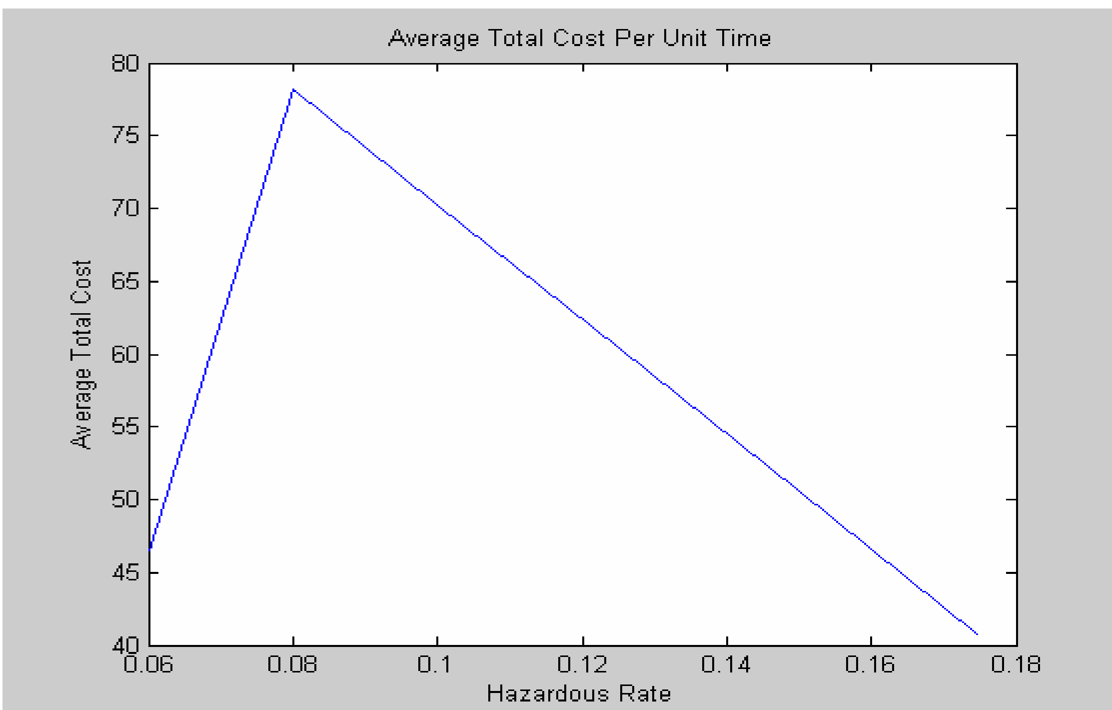
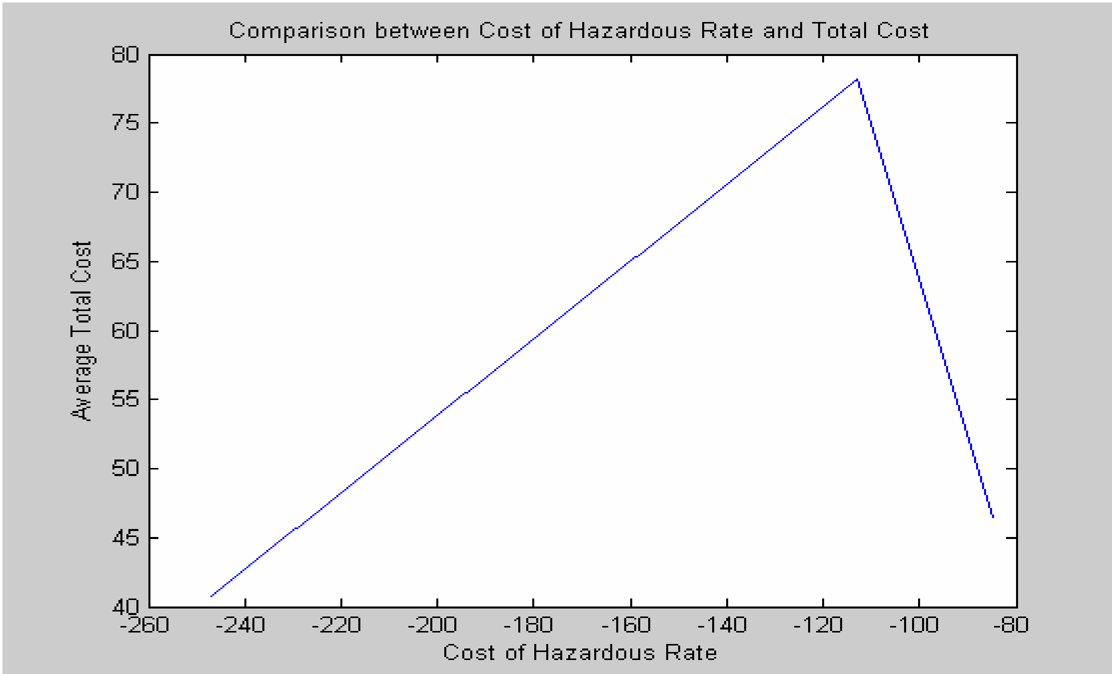


Figure-5



5. Conclusions

The optimal inventory control problem with exponential distributed hazardous item is studied and the mathematical model is developed for optimal purchase quantity. The average total cost per unit time is derived and is found be a relatively simple expression. However, this study minimizes the total cost for constant hazardous rate of exponential distribution.

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